An Efficient Serial-Loop Strategy for Reliability-Based Robust Optimization of Electromagnetic Design Problems

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An efficient reliability-based robust design optimization (RBRDO) is proposed to substantially improve computational efficiency without sacrificing numerical accuracy when applied to electromagnetic (EM) design problems in the presence of uncertainties. Unlike a conventional parallel-loop RBRDO, the robust optimization and reliability assessment are totally decoupled from each other during the overall design process. To ensure the prescribed design feasibility, constraint boundaries in an original RBRDO formulation are first shifted to feasible directions based on the reliability information obtained at a previous design cycle. Then, the first two statistical moments, mean and variance, of a quality loss function is estimated by the univariate dimension reduction method (DRM). After all, the proposed serial-loop methodology employs sequential cycles of an equivalent robust design optimization with shifting constraint boundaries. A simple mathematical model and a BLDC motor design problem are provided to demonstrate the features of the proposed method.

***Index Terms*—Electromagnetics, optimization, reliability theory, robustness.**

I. Introduction

In resent years, industrial demand for ensuring the product quality as well as the confidence in product reliability of EM devices under uncertainties is growing in our community. In response to the requirement, various attempts have been made and applied to EM design problems [1]-[4]. Among them, the robust design optimization (RDO) is to optimize the mean of a product quality loss function and to minimize its variance simultaneously [2]. On the other hand, the reliability-based design optimization (RBDO) focuses on enhancing the confidence in product reliability at a given probabilistic level [3]. However, there have been a few articles to deal with a RBRDO method which integrates two different methodologies mentioned above. In addition, it has been reported that the conventional parallel-loop RBRDO needs an extremely high computational cost when compared to RDO and RBDO [4]. To significantly improve numerical efficiency of RBRDO while maintaining its design accuracy, this paper proposes an efficient serial-loop optimization strategy. In the method, a univariate DRM for RDO is incorporated with a constraint boundary movement technique to secure the design feasibility required for RBDO.

II. RBRDO Strategies

For better understanding, the conventional RBRDO with the parallel-loop structure is briefly summarized. Then a basic concept of the proposed serial-loop method is explained

A. Parallel-Loop Strategy

A typical RBRDO formulation is given by

$$\begin{aligned} &\text{minimize} && f(\mu_h, \sigma_h^2), \quad h(\mathbf{x};\mathbf{d}) \\ &\text{subject to} && P_f(g_i(\mathbf{x})>0) \le P_{t,i}, \quad i=1,2,\dots nc. \end{aligned} \tag{1}$$

where f is a quality loss function consisting of two statistical moments, mean μ_h and variance σ_h^2, of the cost function h, and $\mathbf{d}$ is the vector of design variables defined by $\mathbf{d}=\mu(\mathbf{x})$, where μ denotes the mean of a random vector $\mathbf{x}$ in X-space. The symbol g_i is the ith function of nc constraints, $P_f(\cdot)$ is the probability of failure for the infeasible condition ($g_i > 0$), and $P_{t,i}$ is the ith target value for ensuring the confidence level (1-$P_{t,i}$) of g_i. After all, the goal of RBRDO lies in finding a most insensitive design to the variation of random variables while making probabilistic constraint conditions satisfied at a desired confidence level. Such nested optimization problem requires a parallel-loop optimization structure: to find an optimum of the quality loss function as a main loop, and also to satisfy the given probabilities of constraints as a sub-loop. It may often lead to an unaffordable computational cost when seeking an optimum of (1).

B. Proposed Serial-loop Strategy

To resolve the aforementioned difficult, (1) is converted to an equivalent serial-loop optimization model as follows:

$$\begin{aligned} &\text{minimize} && f(\mu_h, \sigma_h^2), \quad h(\mathbf{x};\mathbf{d}) \\ &\text{subject to} && g_i(\mathbf{x}-\mathbf{s}^k) \le 0, \quad i=1,2,\dots nc \\ & && \mathbf{s}^k = \mu(\mathbf{x}^{k-1}) - \mathbf{x}_{\text{MPP}}^{k-1} \quad (\mathbf{s}^1=0,\ k\ge 2) \end{aligned} \tag{2}$$

where $\mathbf{s}^k$ is a movement direction vector for shifting the constraint boundary (g_i=0) toward a feasible region ($g_i \le 0$) at the kth design cycle, and $\mu(\mathbf{x}^{k-1})$ corresponds to the RDO optimum, $\mathbf{d}^{k-1}$, at the previous design cycle. The symbol $\mathbf{x}_{\text{MPP}}^{k-1}$ means the inverse most probable failure point (MPP) obtained from reliability assessment at the previous design cycle. Instead of solving probabilistic constraints in (1), the proposed model adopts moving constraint boundary conditions, which are gradually shifted by $\mathbf{s}^k$ toward a feasible region to ensure the desired design feasibility at each RDO cycle. Therefore, the proposed serial-loop strategy performs sequential design cycles consisting of equivalent RDO problems with moving constraint conditions. Hereby, the reliability assessment is carried out only once per RDO cycle in the proposed optimization strategy [5]. That can make a great contribution to saving a computational cost of RBRDO.

III. RESULTS

In terms of numerical efficiency and accuracy, the proposed method is verified with the existing parallel-loop RBRDO method. One of the first-order reliability analysis methods called performance measure approach is used to evaluate the probability of failure of a constraint function.

For the first example, a two-dimensional mathematical design problem of (3) is tested.

$$\begin{aligned} &\text{minimize} \quad f(\mu_h,\sigma_h^2) = w_1(\mu_h/\mu_{h0})^2 + w_2(\sigma_h^2/\sigma_{h0}^2)^2 \\ &\qquad\qquad h(\mathbf{x}) = (x_1-4)^3 + (x_1-3)^2 + (x_2-5)^2 + 10 \\ &\text{subject to} \quad P_f(g_1(\mathbf{x}) = -x_1 - x_2 + 7.45 > 0) \le 0.13\% \\ &\qquad\qquad P_f(g_2(\mathbf{x}) : x_i < 1 \text{ or } x_i > 10) \le 0.13\% \quad i = 1,2 \end{aligned} \tag{3}$$

where w_1 and w_2 are weight factors of 0.5 and 1.0 respectively. The symbols, μ_{h0} and σ_{h0}^2, are the nominal values of the cost function h calculated at an initial point (8, 8). The random variable vector $\mathbf{x}$ is assumed to comply with the normal probability distribution with a standard deviation of 0.4. The target probability of failure is set to be 0.13% for two constraints (i.e. reliability of 99.87%)

Lunching at the same initial point, the problem was solved in accordance with two different RBRDO formulations of (1) and (2). The performance indicators of four different designs are compared with each other in Table I. Therein, the RDO optimum was naturally obtained from the first design cycle of the proposed method without the constraint boundary movement. It is observed that although RDO has the smallest value of f, the failure probability of g_1 reaches to more than 50%. Meanwhile, both two RBRDO almost converge into one point, and also yield satisfactory results. In terms of computational efficiency, the proposed method requires the smallest function evaluations even though it has the largest iterative designs. The proposed method reduces the total computational cost by more than 51% compared with the parallel-loop method.

TABLE I
PERFORMANCE INDICATORS AT FOUR DIFFERENT DESIGNS

	Initial	RDO	RBRDO parallel-loop	RBRDO serial-loop
$d_1=\mu(x_1)$	8.00	2.50	3.531	3.529
$d_2=\mu(x_2)$	8.00	5.00	5.673	5.672
Quality loss function f	1.50	0.002	0.005	0.005
$P_f(g_1)$	0%	50.06%	0.12%	0.12%
$P_f(g_2)$	0%	0.00%	0.00%	0.00%
Iterations/Function calls	-	8/98	9/358	15(4)/173

P_f was recalculated by Monte Carlo simulation with 500,000 samples, and the number in parenthesis denotes RDO cycles of the proposed method.

For the second example, a 5 kW, 8-pole and 12-slot BLDC motor in Fig. 1 is considered. The design goal is set to minimize the mean of cogging torque magnitude h and its variation simultaneously. Also, the design has to satisfy a probabilistic constraint condition that the average torque T_{avg} is greater than 20 Nm at a rated speed of 2,300 rpm. To achieve these requirements, a typical RBRDO formulation is written by

$$\begin{aligned} &\text{minimize} \quad f(\mu_h,\sigma_h^2) = w_1(\mu_h/\mu_{h0})^2 + w_2(\sigma_h^2/\sigma_{h0}^2)^2 \\ &\text{subject to} \quad P_F(g(\mathbf{x}) = 20 - T_{avg} > 0) \le 5\% \end{aligned} \tag{4}$$

where w_1 and w_2 are weight factors of 0.5 and 1.0 respectively. Each of three random variables is assumed to follow a normal probability distribution with a standard deviation value of 0.1.

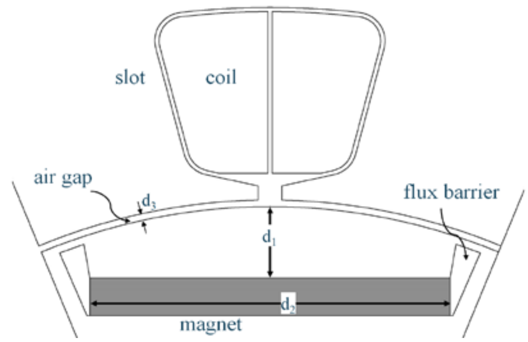

Fig. 1. One-eighth model and three design variables.

A commercial EM simulator called MagNet VII was used to predict the cogging torque with no load condition and average torque at the rated speed [6]. To relieve a computation burden of RBRDO, a deterministic design optimization (DDO) corresponding to (4) was first conducted. Starting with the DDO optimum as an initial design point, two different RBRDO methods were executed separately. The performance indicators between four different designs are presented in Table II. It is obvious that both two RBRDO produce almost same performance indicators of f and T_{avg} when engaged in the randomness of design variables. However, the number of EM simulations of the proposed method is smaller by nearly 28% than that of the parallel-loop method. Numerical results show that the computational efficiency of the proposed method increases in proportion to the number of probabilistic constraint functions.

More detailed explanation and comparative results will be presented in the extended version of the paper.

TABLE II
PERFORMANCE INDICATORS AT FOUR DIFFERENT DESIGNS

	DDO	RDO	RBRDO parallel-loop	RBRDO serial-loop
$d_1=\mu(x_1)$ (mm)	5.548	4.472	4.790	4.792
$d_2=\mu(x_2)$ (mm)	36.407	36.434	37.898	37.895
$d_3=\mu(x_3)$ (mm)	0.711	0.710	0.710	0.707
Quality loss function f	1.5	0.849	0.881	0.880
T_{avg} (Nm)	20.06	20.41	20.82	20.82
Iterative designs	9	5	7	11(3)
Simulator calls	115	235	597	428

The number in parenthesis denotes RDO cycles of the proposed method

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